# Problem 2
# Particular Matrices

The determinant of $M$ would be $det(A \cdot \{0\}) - det(T)$, since $A$ and $T$ commute. The determinant of $A \cdot \{0\}$ is 0. So the only thing left to do is to calculate $det(T \cdot T)$..
We could calculate the determinant of a tridiagonal matrix

$$\begin{vmatrix} a_1 & b_1 & & & \\ c_1 & a_2 & b_2 & & \\ & c_2 & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ & & & c_{n-1} & a_n \end{vmatrix}$$

by using the identity

$$f_n = a_n f_{n-1} - c_{n-1} b_{n-1} f_{n-2}.$$

You can calculate the first Laplace expansion to note recursion and then use some computer code to find determinant for test matrices (which are big).

We could calculate the inverse of a tridiagonal matrix using the identity

$$(T^{-1})_{ij} = \begin{cases} (-1)^{i+j} b_i \cdots b_{j-1} \theta_{i-1} \phi_{j+1} / \theta_n & \text{if } i < j \\ \theta_{i-1} \phi_{j+1} / \theta_n & \text{if } i = j \\ (-1)^{i+j} c_j \cdots c_{i-1} \theta_{j-1} \phi_{i+1} / \theta_n & \text{if } i > j \end{cases}$$

where $\theta$ and $\phi$ satisfy

$$\theta_i = a_i \theta_{i-1} - b_{i-1} c_{i-1} \theta_{i-2}$$
$$\phi_i = a_i \phi_{i+1} - b_i c_i \phi_{i+2}.$$

This is computable in linear time, in contrast with algorithms for general matrices.

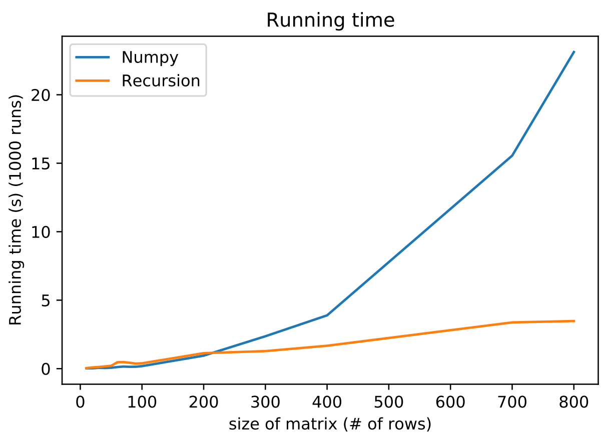


Figure 1: Linear time algorithm.